

RMRRT: Riemannian Barrier Metric RRT for Inequality-Aware Steering on Equality Manifolds

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Abstract—This paper presents a motion planning framework that unifies equality and inequality constraints into a single geometric formulation for sampling-based planning in high-dimensional robotic systems. Conventional sampling-based planners often exhibit inefficient exploration because they treat constraints separately: equality constraints via projection and inequality constraints via binary collision checking. To address this issue, we propose Riemannian Barrier Metric RRT (RMRRT), which unifies both constraints into a single geometric formulation. RMRRT constructs a Riemannian barrier metric on the tangent space of the equality manifold by integrating the null-space projection of the constraint Jacobian with the Hessian of a logarithmic barrier function. This metric guides the tree expansion by selecting steering directions that minimize the induced Riemannian norm, thereby naturally avoiding inequality constraints while strictly adhering to the equality manifold. Experimental results demonstrate that RMRRT achieves a 100% success rate across diverse constrained manipulation tasks in both simulation and real-world experiments, while reducing planning time relative to state-of-the-art constrained planners. Ablation studies further show that the proposed barrier metric reduces sample rejection and shortens path length. Experiment videos and source code are available at: <https://rcilab.github.io/rmrtr>

Index Terms—Constrained motion planning, Sampling-based planning, Riemannian metric, Barrier function, Multi-arm manipulation.

I. INTRODUCTION

SAMPLING-based motion planning becomes particularly challenging when robotic systems are subject to simultaneous equality and inequality constraints [1]–[3]. Equality constraints commonly arise from closed-chain kinematics [4]–[6], grasp maintenance [7]–[9], or cooperative multi-arm manipulation [10], [11], restricting feasible configurations to an implicit manifold embedded in the ambient configuration space [12]–[14]. Meanwhile, inequality constraints—such as obstacle avoidance [15], [16], self-collision [17]–[19], and joint limits [2], [20]—further restrict admissible regions through state-dependent boundaries. Under these conditions, sampling-based planners must efficiently explore admissible regions occupying only a small fraction of the configuration space,

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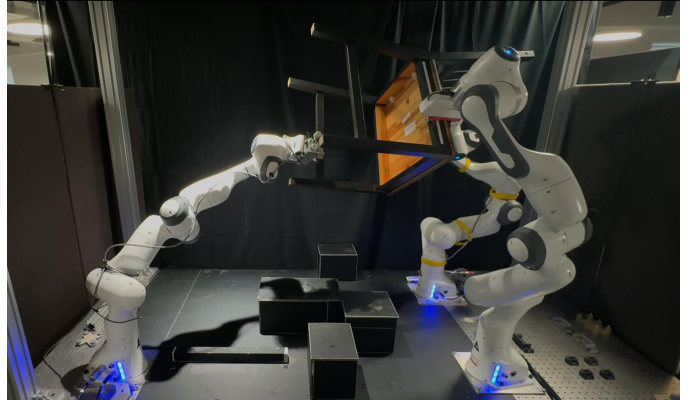


Fig. 1: Real-world demonstration of RMRRT with three manipulators cooperatively transporting a chair under simultaneous equality and inequality constraints for closed-chain grasp maintenance and collision avoidance.

making exploration efficiency and steering quality critical to planning performance.

In conventional sampling-based planners, equality constraints are typically enforced through projection onto the constraint manifold, while inequality constraints are handled separately via binary feasibility checking such as collision testing [12]–[14], [21]. This decoupled treatment can lead to inefficient sample rejection and unstable steering behavior, particularly in highly constrained environments where admissible regions are narrow or fragmented. Consequently, integrating equality and inequality constraints—whose geometric characteristics are inherently distinct—into a unified exploration framework remains a central challenge in sampling-based motion planning. In this work, we address this challenge by proposing a unified geometric formulation that directly encodes inequality-induced boundary geometry into the exploration process on equality-constrained manifolds.

A. Related Works

Sampling-based robot motion planning under constraints has been studied from multiple perspectives, largely distinguished by how equality and inequality constraints are represented and integrated into the exploration process. Prior work can be broadly categorized into three groups: (i) manifold-based approaches that explore directly on equality-constrained manifolds, (ii) metric-based approaches that encode inequality-induced geometry into distance or metric structures, and (iii) data-driven approaches that accelerate sampling and steering using learned or preconstructed priors.

1) *Manifold-Based Motion Planning*: Manifold-based motion planning explores directly on implicitly defined equality-constrained manifolds, aiming to maintain equality feasibility throughout the planning process [5], [12]–[14]. Atlas-based planners approximate the manifold using a collection of local charts and perform global exploration by transitioning across neighboring charts during tree expansion, thereby improving numerical robustness even in highly curved regions [13], [22]. Tangent-space approaches, in turn, generate steering directions in the Jacobian-defined local tangent space and apply manifold-consistent correction (e.g., local projection) to produce equality-consistent extensions [12], [14]. However, in many implementations, tangent-space information is used primarily to ensure equality-feasible extensions, whereas nearest-node selection and steering criteria are not consistently informed by inequality-induced local geometry. Consequently, inequality constraints often influence exploration only through post-hoc rejection via external binary validity checks such as collision testing [23], which can lead to many unproductive extension attempts in narrow passages or heavily cluttered environments [24], [25].

2) *Metric-Based Motion Planning*: Metric-based approaches incorporate inequality constraints into costs, distance measures, or position-dependent metrics to discourage motions near constraint boundaries and bias exploration toward admissible regions. For example, artificial potential fields and continuous cost-based formulations encode obstacle proximity in a smooth and differentiable manner [26], [27], and more recent Riemannian formulations construct non-Euclidean, state-dependent metrics using curvature or Hessian information associated with inequality constraints [28], [29]. Relatedly, reactive motion policy frameworks such as RMPflow [30] compose task-space Riemannian motion policies to generate real-time local control commands under multiple objectives. While effective for reactive behaviors, these methods are not designed to perform sampling-based global exploration on implicit equality-constrained manifolds, and typically require additional mechanisms to strictly enforce closed-chain or grasp-maintenance constraints. Moreover, many metric-based approaches are formulated in unconstrained configuration spaces or incorporate equality constraints only through soft penalties; as a result, additional projection or correction steps are commonly required to ensure consistent steering and tree expansion on equality-constrained manifolds.

3) *Data-driven Motion Planning*: Data-driven approaches accelerate constrained planning by capturing the structure of the feasible space through data or preconstructed priors, thereby biasing sampling distributions and expansion decisions toward favorable regions. In particular, prior work has explored learning low-dimensional latent representations of equality-constrained manifolds [6], [31] to reduce the effective search dimension, as well as using learned samplers or policies to guide exploration. Complementary to learning-based priors, offline structure-based methods such as precomputed graph (roadmap) approaches [5], [32] improve online performance by leveraging roadmaps constructed in advance from constraint-satisfying states to reduce the online search burden. However, the effectiveness of these data- or structure-driven techniques can be sensitive to data coverage or to the density and

suitability of the precomputed structures; moreover, maintaining comparable performance under new environments or changed constraint conditions is difficult to guarantee, and extending the applicability range may require additional data collection or structure updates.

B. Overview of Our Approach

As discussed above, manifold-based planners maintain equality feasibility but handle inequality constraints only through post-hoc rejection, metric-based approaches encode inequality geometry but lack strict equality-manifold consistency, and data-driven methods rely on environment-specific priors that may not generalize. To address these limitations, we propose Riemannian Barrier Metric RRT (RMRRT), a sampling-based planner that enables inequality-aware steering while preserving feasibility on an equality-constrained manifold. The central idea is to endow equality-consistent motions with a Riemannian barrier metric that explicitly encodes the local geometry induced by inequality constraints (e.g., obstacle avoidance, self-collision, and joint limits). Concretely, we construct a state-dependent metric by combining the null-space projection associated with the equality-constraint Jacobian and the Hessian of a logarithmic barrier function defined over inequality constraints, yielding a unified geometric structure for exploration on the manifold.

Based on this metric, RMRRT selects expansion directions by approximately minimizing the induced Riemannian norm within the equality-consistent tangent space, thereby naturally discouraging motions toward inequality boundaries already at the steering stage. The same metric also provides a geometry-consistent notion of local connectability between states, and is therefore leveraged in nearest-node selection to prioritize nodes that admit productive manifold-consistent progress. Consequently, inequality constraints are no longer treated solely as a post-hoc validity filter; instead, tree growth is directly guided along admissible regions, reducing collision-driven rejections and improving exploration efficiency in highly constrained environments. In this work, we instantiate the proposed metric within a bidirectional RRT-Connect framework [33], resulting in a planner that remains compatible with standard sampling-based planning pipelines while preserving feasibility on the equality-constrained manifold and explicitly accounting for inequality constraints during both selection and steering.

The main contributions of this paper are summarized as follows:

- We propose a Riemannian barrier metric that unifies equality-manifold structure and inequality-induced boundary geometry within a single formulation.
- Building on this metric, we develop a bidirectional RRT-based planner that performs inequality-aware exploration while maintaining feasibility on equality-constrained manifolds.
- We integrate the proposed geometry consistently into both steering and nearest-node selection, guiding expansion away from infeasible regions and reducing validity-driven sample rejection during tree growth.

- We validate the proposed method through extensive simulation and real-world experiments on constrained manipulation tasks, including multi-arm and humanoid systems, demonstrating improved planning efficiency and reliability compared to existing constrained planners [6], [13], [14], [21], [32].

The remainder of this paper is organized as follows. Section II introduces the necessary preliminaries, including equality-constrained manifolds and barrier-based metrics. Section III presents the proposed RMRRT methodology in detail. Section IV reports experimental results on a variety of constrained manipulation scenarios. Finally, Section V concludes the paper and discusses directions for future work.

II. PRELIMINARIES

This section summarizes the notation and geometric tools required to develop the proposed method. Specifically, we formalize the motion planning problem under equality and inequality constraints, review the geometric structure of equality-constrained manifolds and their associated tangent spaces, and summarize barrier-based representations for inequality constraints, which will be used to construct the proposed Riemannian barrier metric in Section III.

A. Problem Formulation

We consider motion planning problems for robotic systems operating in a continuous configuration space $\mathcal{Q} \subset \mathbb{R}^n$, where a configuration is denoted by $q \in \mathcal{Q}$. The system is subject to both equality and inequality constraints, which jointly define the feasible configuration space. Equality constraints are expressed as

$$g(q) = 0, \quad (1)$$

where $g : \mathbb{R}^n \rightarrow \mathbb{R}^m$ is a differentiable constraint function. These constraints restrict feasible configurations to an implicitly defined manifold embedded in the ambient configuration space.

In addition, inequality constraints are given by

$$h_i(q) > 0, \quad i = 1, \dots, k \quad (2)$$

where each $h_i : \mathbb{R}^n \rightarrow \mathbb{R}$ represents a state-dependent constraint. Together, the inequality constraints define admissible regions within the equality-constrained manifold. The overall feasible configuration set is therefore defined as

$$\mathcal{F} = \{q \in \mathcal{Q} \mid g(q) = 0, h_i(q) > 0, \forall i\}. \quad (3)$$

We assume that valid start and goal configurations lie within $\mathcal{F}$, and focus on planning collision-free motions that remain feasible throughout the planning process. In sampling-based motion planning, this requires generating tree expansions that preserve equality constraints while avoiding violations of inequality constraints.

B. Equality-Constrained Manifolds and Tangent Spaces

Equality constraints of the form $g(q) = 0$ define an implicit manifold embedded in the ambient configuration space $\mathcal{Q}$. Specifically, the equality-constrained manifold is given by

$$\mathcal{M} = \{q \in \mathcal{Q} \mid g(q) = 0\}. \quad (4)$$

Let $J(q)$ denote the Jacobian of the constraint function,

$$J(q) = \frac{\partial g(q)}{\partial q}. \quad (5)$$

If $J(q)$ has full row rank m for all $q \in \mathcal{M}$, then $\mathcal{M}$ is a smooth manifold of dimension $n - m$ ¹.

At a configuration $q \in \mathcal{M}$, the tangent space of $\mathcal{M}$ is defined as

$$T_q\mathcal{M} = \{v \in \mathbb{R}^n \mid J(q)v = 0\}. \quad (6)$$

This space characterizes the set of infinitesimal velocities that preserve the equality constraints to first order. In sampling-based motion planning, tangent directions are used to construct local expansions, and a retraction is applied when needed to keep the expanded state on the manifold $\mathcal{M}$. Moreover, when $J(q)$ has full row rank m , the matrix $J(q)J(q)^\top$ is invertible, which allows defining the orthogonal projector onto $T_q\mathcal{M}$ as

$$Q(q) = I - J(q)^\top (J(q)J(q)^\top)^{-1} J(q), \quad (7)$$

where I denotes the identity matrix. The projector $Q(q)$ satisfies $J(q)Q(q) = 0$ and maps arbitrary vectors in $\mathbb{R}^n$ onto the tangent space $T_q\mathcal{M}$. This tangent-space formulation provides a fundamental tool for constructing constraint-consistent steering and expansion operators, which will be exploited in the proposed approach.

C. Inequality Constraints and Log-Barrier Functions

As introduced in (2), the inequality constraints induce state-dependent boundaries that restrict admissible configurations within the equality-constrained manifold. In sampling-based planning, such constraints are typically handled via binary feasibility checks. To obtain a continuous representation of inequality constraints, we adopt a log-barrier formulation. Specifically, given a set of k inequality constraints $\{h_i(q)\}_{i=1}^k$, we define the logarithmic barrier function

$$\phi(q) = - \sum_{i=1}^k \log h_i(q). \quad (8)$$

This function grows rapidly as a configuration q approaches the boundary of the admissible region, thereby smoothly encoding proximity to inequality constraints. Moreover, the Hessian of the barrier function captures second-order information associated with the curvature of the constraint boundaries, providing a principled means to quantify how inequality constraints locally distort the geometry of the configuration space. This property makes log-barrier functions well-suited for defining geometry-aware structures that can be leveraged during planning, as will be described in Section III.

III. RIEMANNIAN BARRIER METRIC RRT

In this section, we present the proposed Riemannian Barrier Metric RRT (RMRRT) which is a sampling-based planning framework that unifies equality and inequality constraints through a single geometric formulation. Building on the equality-manifold structure, tangent-space representation, and log-barrier modeling introduced in Section II, RMRRT modifies the local expansion mechanism of RRT planners so that

¹This condition corresponds to the standard regularity assumption of the implicit function theorem.

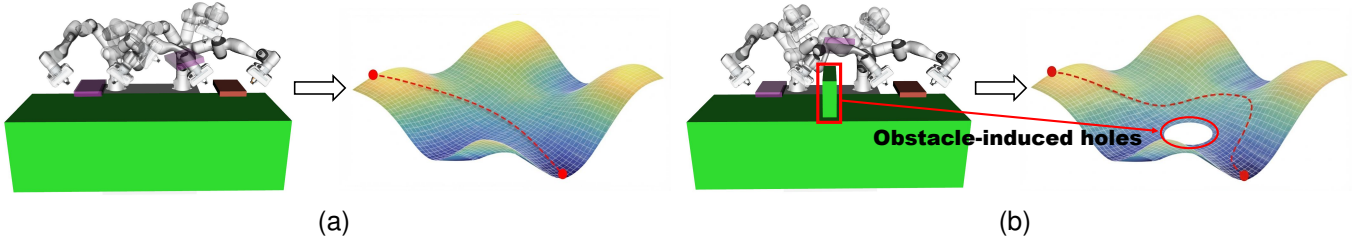


Fig. 2: Conceptual illustration of inequality-induced geometry on an equality-constrained manifold. (a) Equality-constrained manifold under the implicit assumption of full admissibility, as commonly treated in tangent-space and projection-based planners. (b) The same manifold after removing infeasible regions induced by inequality constraints, which form holes in the feasible subset. Although the equality constraint is unchanged, the admissible region exhibits nontrivial geometry that is not captured by conventional steering. RMRRT accounts for this geometry through a Riemannian metric defined on the tangent space.

Algorithm 1 BUILDMETRIC(q): Riemannian barrier metric

Require: Configuration q

Ensure: Inequality metric $G(q)$, tangent projector $P(q)$, Riemannian metric $M(q)$

1: **for** $j = 1, \dots, m$ (**proxy points**) **do**

2: Compute proxy point $p_j(q)$ and signed distance $d_j(q)$

3: Compute point Jacobian $J_j(q) = \partial p_j(q) / \partial q$

4: **end for**

5: $G(q) \leftarrow \lambda I + \sum_{j=1}^m w(d_j(q)) J_j^\top(q) J_j(q)$ (App. A)

6: Compute equality Jacobian $J(q)$

7: $A \leftarrow J(q) G^{-1}(q) J^\top(q)$

8: $P(q) \leftarrow I - G^{-1}(q) J^\top(q) A^{-1} J(q)$ (App. B)

9: $M(q) \leftarrow P^\top(q) G(q) P(q)$

10: **return** $G(q), P(q), M(q)$

exploration explicitly reflects constraint-induced geometry. The remainder of this section describes the overall framework, the construction of the Riemannian barrier metric, and its integration into a bidirectional RRT-Connect planning algorithm.

A. Proposed Framework

The proposed RMRRT builds on a bidirectional RRT-Connect backbone and modifies the local expansion mechanism to account for constraint-induced geometry on the equality-constrained manifold $\mathcal{M}$. To motivate this modification, Fig. 2a illustrates an idealized view in which no regions are carved out by inequality constraints, under which steering is commonly adopted in tangent-space and projection-based approaches [5], [12], [13]. In contrast, Fig. 2b shows that inequality constraints create infeasible regions on the same manifold $\mathcal{M}$, making the feasible subset nontrivial. In such environments, tangent-space steering in $T_q\mathcal{M}$ alone repeatedly attempts expansions toward infeasible regions, leading to increased rejection rates and unstable exploration. To mitigate this issue, RMRRT discourages motions toward inequality boundaries via the Riemannian barrier metric, thereby biasing local expansions along feasible regions. The following subsections describe the metric construction and its integration into the planning pipeline.

B. Riemannian Barrier Metric Construction

Leveraging the barrier-based formulation of inequality constraints introduced in (8), we construct a Riemannian metric that encodes how proximity to inequality boundaries reshapes the local geometry of the configuration space. In principle, the inequality metric is defined by accumulating the exact Hessians of the barrier functions:

$$G(q) = I + \sum_{i=1}^k \alpha_i \nabla^2 \phi_i(q), \quad (9)$$

where ϕ_i denotes the log-barrier associated with the i -th inequality constraint and $\alpha_i > 0$ is a weighting factor. However, in cluttered manipulation scenes, $\phi_i(q)$ typically depends on signed distances and exhibits frequent closest-feature changes, making direct evaluation of $\nabla^2 \phi_i(q)$ computationally expensive and potentially numerically fragile.

To enable efficient and stable metric updates during planning, we therefore adopt a Gauss–Newton (GN) Jacobian-based surrogate that preserves the essential effect of penalizing motions toward inequality boundaries while avoiding explicit second-order derivatives. Specifically, we introduce a set of geometric proxy points $p_j(q) \in \mathbb{R}^3$ on the robot and the manipulated object, together with their signed distances to the environment $d_j(q)$ and point Jacobians $J_j(q)$. The inequality metric is then constructed in a PSD outer-product form as

$$G(q) = \lambda I + \sum_{j=1}^m w(d_j(q)) J_j(q)^\top J_j(q), \quad (10)$$

where $\lambda > 0$ is a small regularizer ensuring positive definiteness and improved conditioning, and $w(d) \geq 0$ is a distance-dependent activation weight that increases as the configuration approaches inequality boundaries. Equation (10) admits a GN interpretation, retaining the dominant term of the barrier Hessian as shown in Appendix A, while leaving the subsequent metric construction unchanged.

Since admissible motions must also satisfy equality constraints, we restrict the metric to the tangent space of the equality-constrained manifold $\mathcal{M}$. Using the equality Jacobian $J(q)$, we define the G -compatible tangent projector

$$P(q) = I - G^{-1}(q) J(q)^\top (J(q) G^{-1}(q) J(q)^\top)^{-1} J(q), \quad (11)$$

whose derivation is provided in Appendix B. The induced

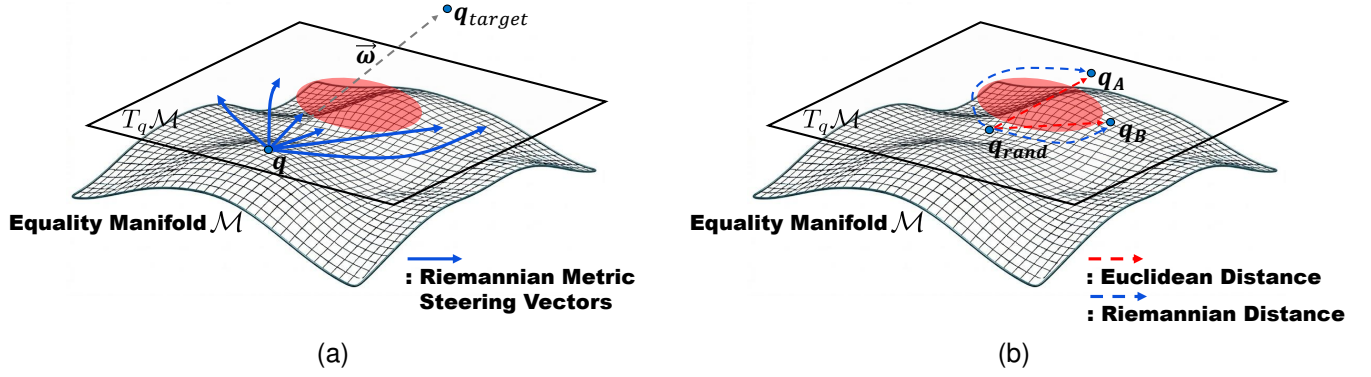


Fig. 3: Conceptual illustration of the Riemannian metric-based exploration mechanism in RMRRT. (a) Metric-induced tangent-space steering. Given a random target, the induced direction is restricted to the tangent space of the equality-constrained manifold at configuration q . Directions that move toward inequality boundaries incur higher cost under the Riemannian barrier metric, thereby biasing steering toward admissible regions. (b) Riemannian metric-based nearest-neighbor selection. Nodes that appear similar in Euclidean space can be separated under the Riemannian distance due to inequality-induced geometry. Consequently, RMRRT preferentially expands nodes that avoid high-cost regions near constraint boundaries, improving exploration stability and expansion efficiency.

Algorithm 2 METRICSTEER($q, q_{\text{target}}, \eta$): Metric-induced tangent-space steering

Require: Feasible node $q \in \mathcal{M}$, target sample q_{target} , step size $\eta > 0$

Ensure: New configuration q_{new}

```

1:  $w \leftarrow q_{\text{target}} - q$ 
2:  $(G, P, M) \leftarrow \text{BUILDMETRIC}(q)$  (Alg. 1)
3:  $\tilde{w} \leftarrow P w$  (tangent projection)
4: if  $\|\tilde{w}\| < \epsilon$  then
5:   return  $q$ 
6: end if
7: Solve  $M v_{\text{tmp}} = \tilde{w}$  (i.e.,  $v_{\text{tmp}} = M^{-1} \tilde{w}$ )
8:  $v_0 \leftarrow P v_{\text{tmp}}$  (re-project for numerical robustness)
9:  $s \leftarrow \sqrt{v_0^\top M v_0}$ 
10: if  $s < \epsilon$  then
11:   return  $q$ 
12: end if
13:  $\hat{v} \leftarrow v_0 / s$  (unit  $M$ -norm direction)
14:  $q_{\text{new}} \leftarrow q + \eta \hat{v}$  (fixed-length step)
15: return  $q_{\text{new}}$ 

```

tangent-space metric is then given by

$$M(q) = P(q)^\top G(q) P(q). \quad (12)$$

All subsequent metric-aware operations, including steering and nearest-neighbor selection, are performed with respect to $M(q)$, and the overall construction of (G, P, M) is summarized in Algorithm 1.

C. Metric-Induced Tangent-Space Steering

Building on the Riemannian barrier metric constructed in Section III-B, we introduce a metric-induced steering rule that

generates local motions on the equality-constrained manifold while respecting the geometry induced by inequality constraints. The key idea is to select, within the tangent space, a direction that minimizes the local energy measured by the induced metric, thereby biasing expansions toward admissible regions, as conceptually illustrated in Fig. 3a. Consider a tree node $q \in \mathcal{M}$ and an ambient target sample q_{target} . The ambient displacement is defined as

$$w = q_{\text{target}} - q. \quad (13)$$

Since a direct step along w generally violates the equality constraints, we first project it onto the tangent space using the metric-compatible projector $P(q)$ defined in (11):

$$\tilde{w} = P(q) w \in T_q \mathcal{M}. \quad (14)$$

Let $G(q) \succ 0$ denote the ambient barrier metric in (10), and let $M(q)$ denote the induced tangent-space metric defined in (12). Among tangent directions, we select the metric-induced steering direction by solving

$$v^* = \arg \min_{v \in T_q \mathcal{M}} \frac{1}{2} v^\top M(q) v \quad \text{s.t.} \quad v^\top \tilde{w} = 1. \quad (15)$$

This optimization biases tangent-space steering away from inequality-boundary directions by assigning higher metric cost to boundary-approaching motions. As derived in Appendix C, (15) admits a closed-form solution, which can be interpreted as reweighting $\tilde{w}$ by the inverse induced metric, yielding a numerically robust tangent direction. The constraint $v^\top \tilde{w} = 1$ fixes the scale only for direction selection; the actual expansion magnitude is controlled separately by the step size in the update rule below. To execute a fixed-length expansion under the induced geometry, we normalize the resulting tangent direction with respect to the $M(q)$ -norm and update the configuration as

$$\hat{v} = \frac{v_0}{\sqrt{v_0^\top M(q) v_0}}, \quad q_{\text{new}} = q + \eta \hat{v}, \quad (16)$$

Algorithm 3 METRICNEAREST($q_{\text{rand}}, \mathcal{V}, K$): metric-aware nearest-neighbor selection

Require: Sample q_{rand} , tree nodes $\mathcal{V} \subset \mathcal{M}$, candidate size K

Ensure: Nearest node q_{near}

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1:  $\mathcal{V}_K \leftarrow \text{EUCLIDEANKNN}(q_{\text{rand}}, \mathcal{V}, K)$ 
2:  $q_{\text{near}} \leftarrow \emptyset, d_{\text{min}} \leftarrow +\infty$ 
3: for all  $q \in \mathcal{V}_K$  do
4:    $(G, P, M) \leftarrow \text{BUILDMETRIC}(q)$  (Alg. 1)
5:    $r \leftarrow q_{\text{rand}} - q$ 
6:    $\tilde{r} \leftarrow P r$ 
7:    $d \leftarrow \sqrt{\tilde{r}^\top M \tilde{r}}$  ( $\|\tilde{r}\|_{M(q)}$ )
8:   if  $d < d_{\text{min}}$  then
9:      $d_{\text{min}} \leftarrow d, q_{\text{near}} \leftarrow q$ 
10:  end if
11: end for
12: return  $q_{\text{near}}$ 
```

where $\eta > 0$ is the step size. Compared to Euclidean tangent-space steering, the proposed rule explicitly encodes inequality-induced geometry: as the configuration approaches inequality boundaries, the barrier metric increases the local energy measured by $M(q)$, thereby suppressing expansions toward infeasible regions. Consequently, the proposed RMRRT planner discourages boundary-seeking motions without relying solely on repeated collision-based rejection, leading to more stable and informative local expansions. The complete procedure is summarized in Algorithm 2.

D. Metric-Aware Sampling and Nearest-Neighbor Selection

In addition to modifying the local steering behavior, the proposed RMRRT framework incorporates the Riemannian barrier metric into sampling and nearest-neighbor selection. Random sampling is performed in the ambient configuration space, as in standard RRT-based planners, in order to preserve the simplicity and exploration properties of conventional RRT sampling. Equality consistency is subsequently enforced during the steering operation through tangent-space projection, as described in Section III-C. Given a sampled configuration q_{rand} and a set of tree nodes $\mathcal{V}$ lying on the equality-constrained manifold $\mathcal{M}$, RMRRT selects the nearest node according to a distance measure induced by the local Riemannian geometry. Specifically, for each node $q \in \mathcal{V}$, we define the metric-aware distance

$$d_M(q, q_{\text{rand}}) = \|P(q)(q_{\text{rand}} - q)\|_{M(q)}, \quad (17)$$

where $P(q)$ denotes the metric-compatible tangent projection and $\|\cdot\|_{M(q)}$ is the norm induced by the Riemannian metric $M(q)$ constructed in Section III-B. In Algorithm 3, this distance is evaluated using the local metric constructed by Algorithm 1 and compared against the current minimum to identify the nearest neighbor under the constraint-induced geometry. The nearest neighbor is then selected as

$$q_{\text{near}} = \arg \min_{q \in \mathcal{V}} d_M(q, q_{\text{rand}}). \quad (18)$$

As conceptually illustrated in Fig. 3b, proximity in the ambient Euclidean configuration space does not necessarily

Algorithm 4 Overall RMRRT Framework

Require: Feasible start/goal configurations $q_{\text{start}}, q_{\text{goal}} \in \mathcal{M}$;

step size η ; candidate size K ; maximum iterations N_{max}

Ensure: Feasible path σ or FAILURE

```

1: Initialize two trees  $\mathcal{T}_a \leftarrow \{q_{\text{start}}\}, \mathcal{T}_b \leftarrow \{q_{\text{goal}}\}$ 
2: for  $n = 1$  to  $N_{\text{max}}$  do
3:    $q_{\text{rand}} \sim \text{UNIFORMSAMPLE}(\mathbb{R}^n)$ 
4:    $q_{\text{near}} \leftarrow \text{METRICNEAREST}(q_{\text{rand}}, \mathcal{V}(\mathcal{T}_a), K)$  (Alg. 3)
5:    $q_{\text{new}} \leftarrow \text{METRICSTEER}(q_{\text{near}}, q_{\text{rand}}, \eta)$  (Alg. 2)
6:    $\text{isStateValid} \leftarrow \text{STATEVALID}(q_{\text{new}})$ 
7:    $\text{isEdgeValid} \leftarrow \text{EDGEVALID}(q_{\text{near}}, q_{\text{new}})$ 
8:   if not  $\text{isStateValid}$  or not  $\text{isEdgeValid}$  then
9:     continue
10:  end if
11:  Add  $q_{\text{new}}$  to  $\mathcal{T}_a$  with parent  $q_{\text{near}}$ 
12:   $q_{\text{curr}} \leftarrow \text{METRICNEAREST}(q_{\text{new}}, \mathcal{V}(\mathcal{T}_b), K)$  (Alg. 3)
13:  repeat
14:     $q_{\text{next}} \leftarrow \text{METRICSTEER}(q_{\text{curr}}, q_{\text{new}}, \eta)$ 
15:     $\text{isStateValid} \leftarrow \text{STATEVALID}(q_{\text{next}})$ 
16:     $\text{isEdgeValid} \leftarrow \text{EDGEVALID}(q_{\text{curr}}, q_{\text{next}})$ 
17:    if not  $\text{isStateValid}$  or not  $\text{isEdgeValid}$  then
18:      break
19:    end if
20:    Add  $q_{\text{next}}$  to  $\mathcal{T}_b$  with parent  $q_{\text{curr}}$ 
21:     $q_{\text{curr}} \leftarrow q_{\text{next}}$ 
22:  until  $\|q_{\text{curr}} - q_{\text{new}}\| < \epsilon_{\text{conn}}$ 
23:  if  $\|q_{\text{curr}} - q_{\text{new}}\| < \epsilon_{\text{conn}}$  then
24:     $\sigma \leftarrow \text{EXTRACTPATH}(\mathcal{T}_a, q_{\text{new}}, \mathcal{T}_b, q_{\text{curr}})$ 
25:    return  $\sigma$ 
26:  end if
27:  Swap  $(\mathcal{T}_a, \mathcal{T}_b)$ 
28: end for
29: return FAILURE
```

imply feasibility on the equality-constrained manifold. Multiple tree nodes may appear close to a sampled configuration q_{rand} , while some of them lie inside infeasible regions formed by inequality constraints. By evaluating distances using the Riemannian barrier metric projected onto the tangent space, RMRRT favors nodes whose local tangent spaces admit feasible, low-cost motions toward the sampled target, while naturally avoiding nodes located inside such infeasible regions. In practice, the metric-aware nearest-neighbor search is restricted to the K Euclidean-nearest candidates in the ambient space for efficiency; see the Appendix D for details.

E. Overall RMRRT Algorithm

Algorithm 4 summarizes the overall RMRRT framework. The proposed method integrates Riemannian barrier metric-based nearest-neighbor selection and tangent-space steering into a bidirectional RRT-Connect structure, thereby preserving the simplicity and scalability of standard sampling-based

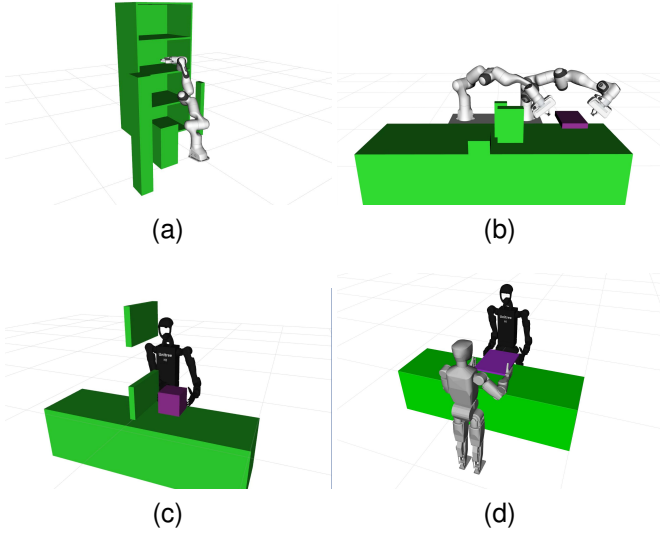


Fig. 4: Representative examples demonstrating the applicability of RMRRT to multiple robotic platforms. (a) single arm manipulator, (b) multi arm manipulators, (c) single humanoid, (d) dual humanoids.

planning while jointly accounting for equality and inequality constraints. RMRRT draws random samples q_{rand} in the ambient configuration space and selects the nearest node q_{near} from the active tree using the metric-aware criterion in Section III-D. It then performs a local expansion using the metric-induced tangent-space steering rule in Section III-C, which generates an equality-consistent local motion while biasing the expansion direction according to the local geometry induced by inequality constraints.

After a candidate state is generated, hard feasibility is enforced through standard state and edge validity checks. If the expansion is valid, the new node is added to the active tree, and the opposite tree is repeatedly extended toward it in a connect phase. In our implementation, the connection seed in the opposite tree is also selected using the metric-aware nearest-neighbor rule, so that both tree expansions consistently reflect the same constraint-induced geometry. The connect phase is repeated until the two trees are connected within ϵ_{conn} or the extension becomes trapped. The local Riemannian barrier metric used by the nearest-neighbor and steering modules is constructed on demand via Algorithm 1. Reusing the same local metric preserves geometric consistency between global exploration and local expansion. In addition, a lightweight projection step may be applied when numerical drift from the equality-constrained manifold is observed. By encoding inequality effects directly into the local geometry, RMRRT remains compatible with existing RRT-Connect pipelines while improving exploration robustness and expansion quality in constrained environments.

IV. EXPERIMENTS

This section evaluates RMRRT under simultaneous equality and inequality constraints to assess its reliability, efficiency, and practical applicability. As shown in Fig. 4, the proposed algorithm is validated across multiple robotic platforms, including

single-arm and multi-arm manipulators as well as single and dual humanoid systems. Simulation benchmarks are presented across diverse manipulation scenarios, including closed-chain, multi-arm, and cluttered environments. Planning performance is quantified using success rate, planning time, and path quality metrics. An ablation study is conducted in a densely cluttered dual-arm setting to isolate the contribution of the Riemannian barrier metric through comparisons between variants with and without the barrier term. Additional real-world experiments and demonstration videos are available on the project page: <https://rcilab.github.io/rmrtr>.

A. Simulation Benchmarks

For the simulation benchmarks, we constructed constrained manipulation scenarios spanning single-arm to triple-arm systems. All simulations were conducted on the same computing platform, equipped with an Intel Core i7-13700F processor (2.1 GHz), 32 GB of RAM, and an NVIDIA GeForce RTX 4060 GPU. For each scenario, 100 trials were performed using the same start state q_{start} and goal state q_{goal} , and a trial was counted as successful if a valid solution was found within 300 s. All methods were evaluated under the same experimental conditions, while each planner was configured individually according to its own internal parameterization. The parameters were tuned based on quantities with analogous roles across methods, such as expansion length, local extension granularity, and manifold-feasibility tolerance.

Table I summarizes planning performance across four scenarios of increasing constraint complexity, and Fig. 5 visualizes the corresponding tasks and representative trajectories. RMRRT achieves a 100% success rate in all scenarios, indicating that the proposed metric-consistent exploration strategy reliably preserves equality constraints while expanding within regions shaped by inequality constraints. Manifold-based planners such as CBiRRT [21], Atlas-RRT [13], and Tangent Bundle RRT [13], [14] also operate on the equality-constrained manifold by constructing local tangent spaces and correcting drift via projection. However, their tangent-space machinery is typically used primarily for local expansion and is not consistently integrated into the overall exploration strategy. In particular, nearest-node selection is commonly performed using ambient Euclidean distance, which does not adequately reflect intrinsic connectability on the constraint manifold, as conceptually illustrated in Fig. 3b.

By contrast, RMRRT incorporates the induced geometry into both selection and steering: the nearest-node selection defined in Section III-D ranks nodes using the Riemannian distance induced by $M(q)$, while the steering method defined in Section III-C performs metric-induced reweighting under the same induced metric. This coupling prioritizes nodes that admit effective equality-consistent motion toward the target and suppresses boundary-seeking motions at an earlier stage, thereby reducing collision-driven rejection, shortening mean planning time, and improving success rate. In addition, the combination of geometry-aware selection and metric-based steering makes successful extensions more uniform across trials, which contributes to a lower standard deviation of planning time.

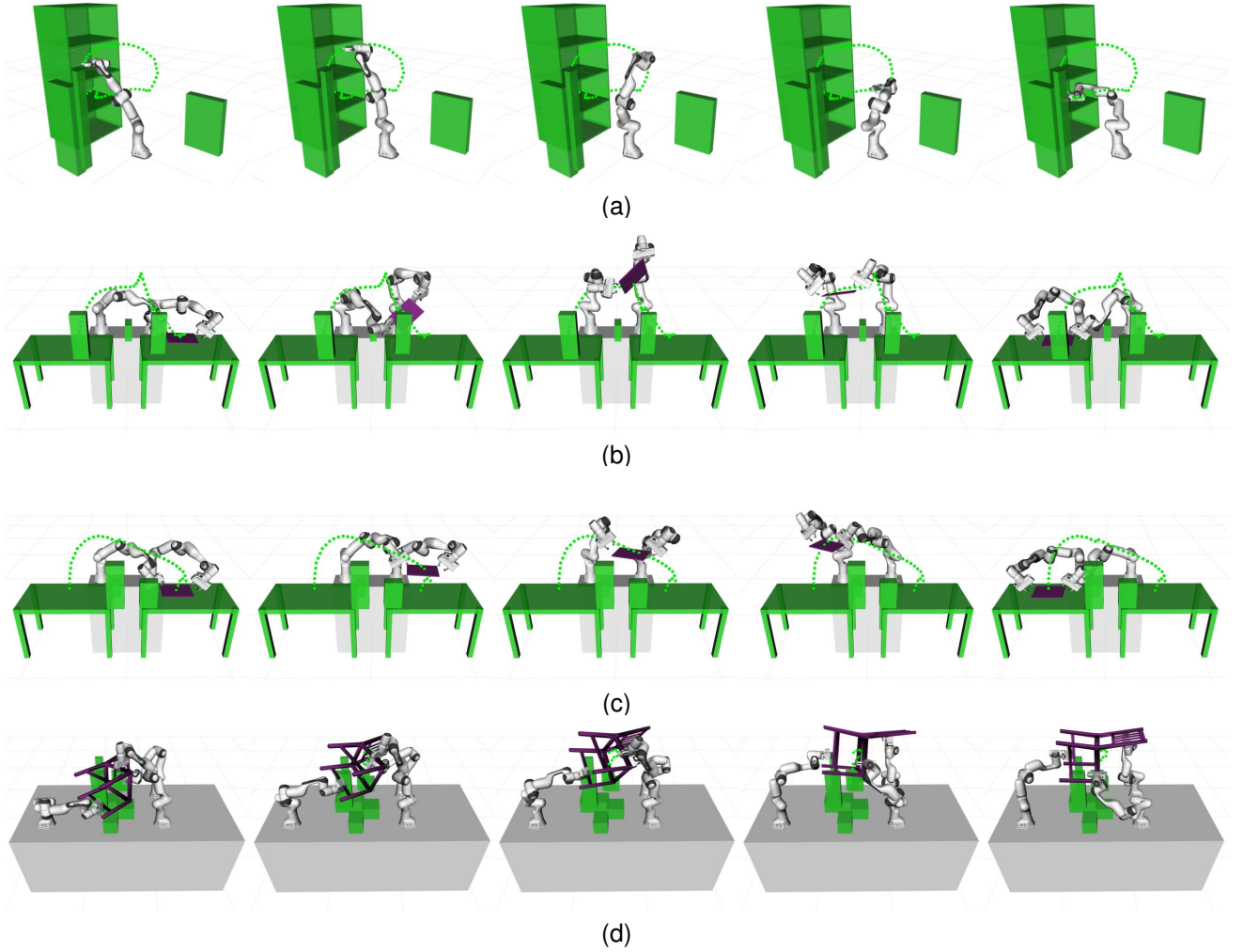


Fig. 5: Planned trajectories in four representative manipulation scenarios. The dotted curve indicates the end-effector (or object) trajectory, and snapshots depict key configurations along each path. (a) Single-arm cabinet reaching with fixed orientation constraint. (b) Dual-arm constrained transfer. (c) Dual-arm transfer with fixed orientation constraint. (d) Triple-arm constrained manipulation.

TABLE I: Planning performance across four experimental scenarios.

Planner	Scenario 1		Scenario 2		Scenario 3		Scenario 4	
	Time (s)	Succ. (%)	Time (s)	Succ. (%)	Time (s)	Succ. (%)	Time (s)	Succ. (%)
CBiRRT [21]	0.35 ± 0.41	100	2.97 ± 4.48	100	3.06 ± 4.27	100	74.75 ± 118.59	90
Atlas RRT [13]	5.84 ± 12.63	100	48.55 ± 85.13	83	35.12 ± 69.27	89	10.02 ± 31.80	89
Tangent Bundle RRT [14]	5.74 ± 16.75	100	37.71 ± 86.93	80	48.83 ± 86.27	87	15.80 ± 57.35	84
Precomputed Graph RRT [32]	2.65 ± 6.64	100	4.49 ± 5.43	100	5.51 ± 10.41	100	32.37 ± 81.47	97
LJCMP [6]	0.41 ± 1.02	100	0.24 ± 0.53	100	0.48 ± 1.56	100	0.55 ± 1.69	100
RMRRT (ours)	0.32 ± 0.34	100	0.18 ± 0.15	100	0.25 ± 0.26	100	0.38 ± 0.51	100

We also compare against Precomputed Graph RRT [32] and LJCMP [6]. As shown in Table I, Precomputed Graph RRT exhibits stable performance in simpler settings but degrades noticeably in Scenario 4, suggesting that a fixed offline structure may not sufficiently capture local geometric variations in high-dimensional constrained spaces. In contrast, LJCMP maintains high success rates and low average planning times across all scenarios, demonstrating the effectiveness of learning-based priors in guiding sampling toward favorable regions.

While LJCMP benefits from learned priors, RMRRT achieves

competitive performance with low variance across all scenarios without relying on learning or precomputed roadmaps, offering a purely geometric alternative. It is worth highlighting that RMRRT can also serve as a geometric enhancement layer: integrating Riemannian geometry into nearest-neighbor selection and steering provides a principled mechanism for improving geometric consistency during exploration, and can be incorporated as a complementary module in learning-based planners to enhance both efficiency and stability.

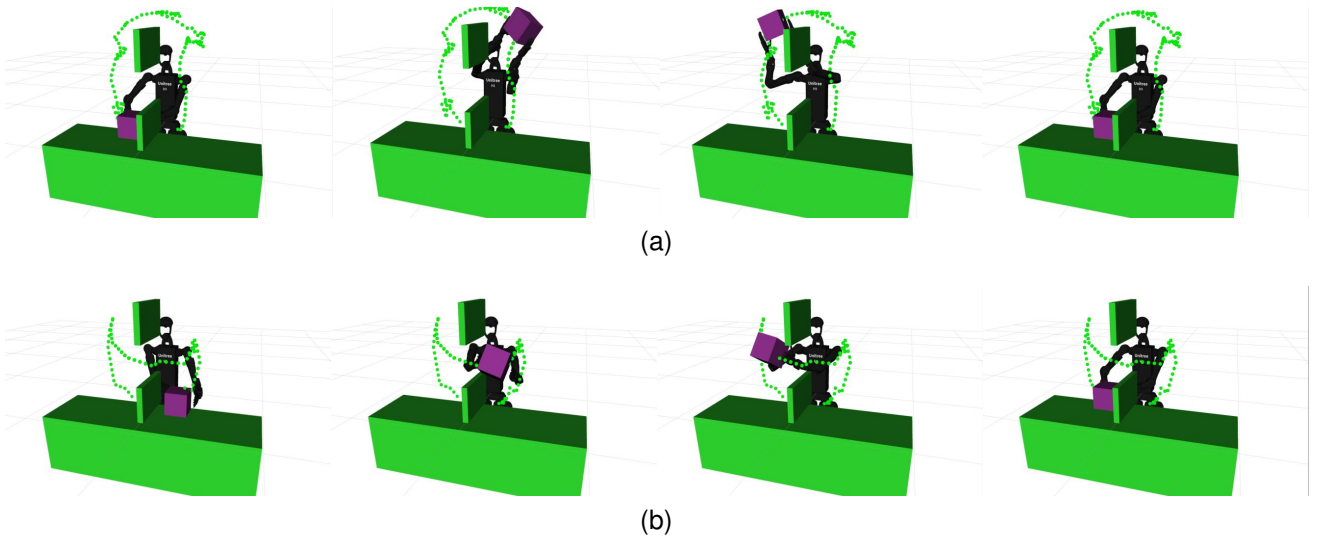


Fig. 6: Ablation study of the proposed Riemannian barrier metric on a narrow-passage constrained transfer task using a Unitree H1-2 humanoid. The figure shows selected trajectories from Scenario 5 to illustrate the effect of the barrier metric in a restrictive passage environment. (a) A trajectory obtained without the barrier metric. (b) A trajectory obtained with the barrier metric.

TABLE II: Ablation study on the effect of the Riemannian barrier metric (Scenario 5).

Method	Path Length (m)	Reject Samples (N)	Tangent Steering Failure (%)
RMRRT (w/o barrier)	31.74 ± 13.49	693.46 ± 312.35	0 ± 0
RMRRT (full)	25.06 ± 7.76	612.2 ± 193.5	0 ± 0

B. Ablation Study on the Riemannian Barrier Metric

To isolate the effect of the proposed Riemannian barrier metric from other components of the planner, we conduct an ablation study on a narrow-passage constrained transfer task using a Unitree H1-2 single-humanoid dual-arm system. In Scenario 5, obstacles are densely arranged so that inequality constraints arising from obstacle avoidance strongly restrict locally admissible directions and passable regions on the equality-constrained manifold. We compare the full RMRRT formulation, which incorporates the Riemannian barrier metric defined in (12), with a variant in which the barrier term is disabled so that the metric no longer reflects the local geometry near inequality boundaries. For a fair comparison, all other components including the sampling strategy, nearest-neighbor selection, steering rule, validity checking, termination conditions, and key hyperparameters, are kept identical. Therefore, the observed differences can be attributed primarily to the presence or absence of the barrier metric.

As shown in Table II, the tangent steering failure rate remains identically zero in both cases, confirming that the underlying framework preserves equality-manifold feasibility regardless of the barrier term. It is worth noting that both variants successfully find solutions in all trials; the barrier metric does not affect equality-constraint satisfaction, but rather improves the quality of exploration by steering away from inequality boundaries. For example, in narrow-passage environments such as shown in Fig. 6, where inequality boundaries strongly govern the local exploration geometry, the effect of the barrier metric becomes more apparent. Specifically, both variants successfully traverse the passage, but the RMRRT

without the barrier term occasionally fails to efficiently navigate through the narrow passage, producing longer and less direct trajectories as illustrated in Fig. 6a, resulting in larger mean and variance in path length as reported in Table II. Moreover, the increase in rejected samples should not be interpreted merely as more collisions, but rather as evidence that locally unproductive expansions are attempted more frequently. When the barrier metric is enabled, the planner traverses constrained regions more directly and consistently, as shown in Fig. 6b, confirming that the barrier metric improves exploration quality by incorporating inequality-induced local geometry into the planning process.

V. CONCLUSION

In this paper, we proposed RMRRT, a sampling-based motion planning framework that enables inequality-aware steering while preserving feasibility on equality-constrained manifolds. By constructing a Riemannian barrier metric on the tangent space of the equality manifold, RMRRT integrates inequality-induced geometry directly into both steering and nearest-node selection, naturally guiding tree expansion away from infeasible regions without relying solely on collision-based sample rejection. The proposed framework was validated through a wide range of manipulation scenarios including multi-arm and humanoid systems, with detailed benchmark comparisons and ablation studies demonstrating its advantages over existing constrained and learning-based planners. Additional real-world experiments and demonstration videos are available on <https://rcilab.github.io/rmrtr>. Across these evaluations, RMRRT consistently demonstrated improved planning efficiency and

reliability compared to existing constrained and learning-based planners.

Future work will explore the integration of the proposed RMRRT framework with learning-based motion planning approaches. Since the proposed steering mechanism is geometrically defined on equality-constrained manifolds and operates independently of assumptions on sampling distributions or latent representations, it has strong potential to complement learning-based planners by enhancing local expansion efficiency when combined with learned sampling strategies, heuristics, or latent-space exploration.

APPENDIX A

GAUSS-NEWTON APPROXIMATION OF THE BARRIER METRIC

This appendix justifies the Jacobian-based ambient metric in (10) as a Gauss-Newton (GN) surrogate of the ideal Hessian accumulation in (9). For a distance-based inequality constraint with signed distance $d(q)$ and barrier $\phi(q) = \phi(d(q))$, the chain rule gives

$$\nabla^2 \phi(q) = \phi''(d) \nabla d(q) \nabla d(q)^\top + \phi'(d) \nabla^2 d(q). \quad (\text{A1})$$

Near closest-feature changes and contact switching, $\nabla^2 d(q)$ can be ill-conditioned or undefined; thus we adopt the GN approximation

$$\nabla^2 \phi(q) \approx \phi''(d) \nabla d(q) \nabla d(q)^\top, \quad (\text{A2})$$

which retains the dominant PSD term. Using a geometric proxy point $p(q) \in \mathbb{R}^3$ with Jacobian $J(q) = \partial p(q)/\partial q \in \mathbb{R}^{3 \times n}$ and distance normal $n(q) = \partial d/\partial p \in \mathbb{R}^3$, we have

$$\nabla d(q) = n(q)^\top J(q), \quad (\text{A3})$$

and substituting (A3) into (A2) yields

$$\nabla^2 \phi(q) \approx \phi''(d) J(q)^\top (n(q) n(q)^\top) J(q) \preceq \phi''(d) J(q)^\top J(q), \quad (\text{A4})$$

since $n(q) n(q)^\top \preceq I$. Absorbing $\phi''(d)$ into a nonnegative weight $w(d) \geq 0$ and summing over m proxy points gives the practical GN barrier metric

$$G(q) = \lambda I + \sum_{j=1}^m w(d_j(q)) J_j(q)^\top J_j(q), \quad (\text{A5})$$

where $\lambda > 0$ ensures positive definiteness and improves conditioning.

APPENDIX B

DERIVATION OF THE METRIC-AWARE PROJECTION OPERATOR

Consider the equality-constrained manifold $\mathcal{M} = \{q \in \mathbb{R}^n \mid g(q) = 0\}$ with Jacobian $J(q) = \partial g/\partial q \in \mathbb{R}^{m \times n}$ and tangent space $T_q \mathcal{M} = \{v \in \mathbb{R}^n \mid J(q)v = 0\}$. Given $w \in \mathbb{R}^n$ and an SPD metric $G(q)$, the G -orthogonal projection of w onto $T_q \mathcal{M}$ is defined by

$$\min_{v \in \mathbb{R}^n} \frac{1}{2} (v - w)^\top G(q) (v - w) \quad \text{s.t.} \quad J(q)v = 0. \quad (\text{B1})$$

The KKT conditions are

$$G(q)(v - w) + J(q)^\top \lambda = 0, \quad J(q)v = 0. \quad (\text{B2})$$

Eliminating v gives

$$\lambda = (J(q)G^{-1}(q)J(q)^\top)^{-1} J(q)w, \quad (\text{B3})$$

and hence $v = P(q)w$ with

$$P(q) = I - G^{-1}(q)J(q)^\top (J(q)G^{-1}(q)J(q)^\top)^{-1} J(q). \quad (\text{B4})$$

By construction, $J(q)P(q) = 0$, thus $P(q)w \in T_q \mathcal{M}$.

APPENDIX C

DERIVATION OF THE METRIC-INDUCED STEERING DIRECTION

Let $w \in \mathbb{R}^n$ be an ambient displacement (e.g., $w = q_{\text{rand}} - q$) and let $\tilde{w} = P(q)w \in T_q \mathcal{M}$. Recall the induced tangent-space metric $M(q) = P(q)^\top G(q)P(q)$ from (12). To obtain a metric-induced tangent direction that minimizes local energy while making progress along $\tilde{w}$, consider

$$v^* = \arg \min_{v \in T_q \mathcal{M}} \frac{1}{2} v^\top M(q) v \quad \text{s.t.} \quad v^\top \tilde{w} = 1. \quad (\text{C1})$$

Using $\mathcal{L}(v, \lambda) = v^\top M v + \lambda(1 - v^\top \tilde{w})$ gives

$$v^* = \frac{M^{-1}(q)\tilde{w}}{\tilde{w}^\top M^{-1}(q)\tilde{w}}. \quad (\text{C2})$$

In implementation, we compute $v_{\text{tmp}} = M^{-1}(q)\tilde{w}$, apply the projector once more for numerical robustness,

$$v_0 = P(q)v_{\text{tmp}} = P(q)M^{-1}(q)\tilde{w} = P(q)M^{-1}(q)P(q)w, \quad (\text{C3})$$

and take a fixed-length step using the unit $M(q)$ -norm direction

$$\hat{v} = \frac{v_0}{\sqrt{v_0^\top M(q) v_0}}, \quad q^+ = q + \eta \hat{v}, \quad (\text{C4})$$

where $\eta > 0$ is the step size.

APPENDIX D

METRIC-AWARE NEAREST-NEIGHBOR EVALUATION

Given a sample $q_{\text{rand}} \in \mathbb{R}^n$ and tree nodes $\mathcal{V} \subset \mathcal{M}$, the metric-aware nearest neighbor is

$$q_{\text{near}} = \arg \min_{q \in \mathcal{V}} \|P(q)(q_{\text{rand}} - q)\|_{M(q)}. \quad (\text{D1})$$

In practice, we restrict evaluation to the K Euclidean-nearest candidates in the ambient space,

$$\mathcal{V}_K = \arg \min_{\substack{\mathcal{S} \subset \mathcal{V} \\ |\mathcal{S}|=K}} \sum_{q \in \mathcal{S}} \|q_{\text{rand}} - q\|_2, \quad (\text{D2})$$

and compute the metric-aware distance only over $\mathcal{V}_K$:

$$q_{\text{near}} = \arg \min_{q \in \mathcal{V}_K} \|P(q)(q_{\text{rand}} - q)\|_{M(q)}. \quad (\text{D3})$$

REFERENCES

- [1] C. Mu, Z. Jiang, J. Zhang, K. Wang, X. Xu, and J. Yi, "Efficient exploration in large state-action space through structured action space for learning multirobots motion planning," *IEEE Transactions on Control Systems Technology*, 2025.
- [2] M. Stilman, "Task constrained motion planning in robot joint space," in *2007 IEEE/RSJ International Conference on Intelligent Robots and Systems*. IEEE, 2007, pp. 3074–3081.
- [3] Z. Xian, P. Lertkultanon, and Q.-C. Pham, "Closed-chain manipulation of large objects by multi-arm robotic systems," *IEEE Robotics and Automation Letters*, vol. 2, no. 4, pp. 1832–1839, 2017.
- [4] R. Borden, L. Ros, and J. M. Porta, "A randomized kinodynamic planner for closed-chain robotic systems," *IEEE Transactions on Robotics*, vol. 37, no. 1, pp. 99–115, 2020.
- [5] K. Jang, J. Baek, S. Park, and J. Park, "Motion planning for closed-chain constraints based on probabilistic roadmap with improved connectivity," *IEEE/ASME Transactions on Mechatronics*, vol. 27, no. 4, pp. 2035–2043, 2022.

- [6] S. Park, S. Jeon, and J. Park, "A constrained motion planning method exploiting learned latent space for high-dimensional state and constraint spaces," *IEEE/ASME Transactions on Mechatronics*, vol. 29, no. 4, pp. 3001–3009, 2024.
- [7] W. Liu, M. Ren, K. Song, M. Y. Wang, and Z. Xiong, "A planning framework for complex flipping manipulation of multiple mobile manipulators," *IEEE Robotics and Automation Letters*, 2025.
- [8] M. Dogar, A. Spielberg, S. Baker, and D. Rus, "Multi-robot grasp planning for sequential assembly operations," *Autonomous Robots*, vol. 43, no. 3, pp. 649–664, 2019.
- [9] S. Zimmermann, G. Hakimifard, M. Zamora, R. Poranne, and S. Coros, "A multi-level optimization framework for simultaneous grasping and motion planning," *IEEE Robotics and Automation Letters*, vol. 5, no. 2, pp. 2966–2972, 2020.
- [10] K. Shi, J. Liu, J. Chang, and G. Hu, "Distributed quadratic programming for safe human multirobot interaction under task constraints," *IEEE/ASME Transactions on Mechatronics*, 2025.
- [11] J. Alonso-Mora, R. Knepper, R. Siegwart, and D. Rus, "Local motion planning for collaborative multi-robot manipulation of deformable objects," in *2015 IEEE international conference on robotics and automation (ICRA)*. IEEE, 2015, pp. 5495–5502.
- [12] C. Suh, T. T. Um, B. Kim, H. Noh, M. Kim, and F. C. Park, "Tangent space rrt: A randomized planning algorithm on constraint manifolds," in *2011 IEEE International Conference on Robotics and Automation*. IEEE, 2011, pp. 4968–4973.
- [13] L. Jaillet and J. M. Porta, "Path planning under kinematic constraints by rapidly exploring manifolds," *IEEE Transactions on Robotics*, vol. 29, no. 1, pp. 105–117, 2012.
- [14] B. Kim, T. T. Um, C. Suh, and F. C. Park, "Tangent bundle rrt: A randomized algorithm for constrained motion planning," *Robotica*, vol. 34, no. 1, pp. 202–225, 2016.
- [15] J. J. Quiroz-Omaña and B. V. Adorno, "Whole-body control with (self) collision avoidance using vector field inequalities," *IEEE Robotics and Automation Letters*, vol. 4, no. 4, pp. 4048–4053, 2019.
- [16] D. Guo and Y. Zhang, "A new inequality-based obstacle-avoidance mvn scheme and its application to redundant robot manipulators," *IEEE Transactions on Systems, Man, and Cybernetics, Part C (Applications and Reviews)*, vol. 42, no. 6, pp. 1326–1340, 2012.
- [17] R. Grassmann, L. Johannismeier, and S. Haddadin, "Smooth point-to-point trajectory planning in $se(3)$ with self-collision and joint constraints avoidance," in *2018 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS)*. IEEE, 2018, pp. 1–9.
- [18] P. Hu, A. Huang, J. Dolan, D. Held, and D. Ramanan, "Safe local motion planning with self-supervised freespace forecasting," in *Proceedings of the IEEE/CVF Conference on Computer Vision and Pattern Recognition (CVPR)*, June 2021, pp. 12 732–12 741.
- [19] K. Cai, L. Zhang, X. Su, K. Chen, C. Wang, S. Haddadin, A. Knoll, A. Ajoudani, and L. Figueredo, "Just-in-time informed trees: Manipulability-aware asymptotically optimized motion planning," *IEEE/ASME Transactions on Mechatronics*, 2025.
- [20] W. Li, Z. Yi, R. Rao, D. Guo, Y. Pan, and K. Huang, "A quadratic programming solution to position-level repetitive motion planning and obstacle avoidance of joint-constrained robots," *IEEE Transactions On Industrial Electronics*, 2024.
- [21] D. Berenson, S. S. Srinivasa, D. Ferguson, and J. J. Kuffner, "Manipulation planning on constraint manifolds," in *2009 IEEE international conference on robotics and automation*. IEEE, 2009, pp. 625–632.
- [22] L. Jaillet and J. M. Porta, "Path planning with loop closure constraints using an atlas-based rrt," in *Robotics Research: The 15th International Symposium ISRR*. Springer, 2016, pp. 345–362.
- [23] Z. Kingston, M. Moll, and L. E. Kavraki, "Sampling-based methods for motion planning with constraints," *Annual review of control, robotics, and autonomous systems*, vol. 1, no. 1, pp. 159–185, 2018.
- [24] D. Hsu, T. Jiang, J. Reif, and Z. Sun, "The bridge test for sampling narrow passages with probabilistic roadmap planners," in *2003 IEEE international conference on robotics and automation (cat. no. 03CH37422)*, vol. 3. IEEE, 2003, pp. 4420–4426.
- [25] J. Pan and D. Manocha, "Fast probabilistic collision checking for sampling-based motion planning using locality-sensitive hashing," *The International Journal of Robotics Research*, vol. 35, no. 12, pp. 1477–1496, 2016.
- [26] O. Khatib, "Real-time obstacle avoidance for manipulators and mobile robots," *The international journal of robotics research*, vol. 5, no. 1, pp. 90–98, 1986.
- [27] N. Ratliff, M. Zucker, J. A. Bagnell, and S. Srinivasa, "Chomp: Gradient optimization techniques for efficient motion planning," in *2009 IEEE international conference on robotics and automation*. IEEE, 2009, pp. 489–494.
- [28] A. Manjunath and Q. Nguyen, "Safe and robust motion planning for dynamic robotics via control barrier functions," in *2021 60th IEEE Conference on Decision and Control (CDC)*. IEEE, 2021, pp. 2122–2128.
- [29] G. Yang, B. Vang, Z. Serlin, C. Belta, and R. Tron, "Sampling-based motion planning via control barrier functions," in *Proceedings of the 2019 3rd International Conference on Automation, Control and Robots*, 2019, pp. 22–29.
- [30] C.-A. Cheng, M. Mukadam, J. Issac, S. Birchfield, D. Fox, B. Boots, and N. Ratliff, "Rmpflow: A geometric framework for generation of multitask motion policies," *IEEE Transactions on Automation Science and Engineering*, vol. 18, no. 3, pp. 968–987, 2021.
- [31] J. Zhang, C. Bai, W. Pan, T. Liu, and J. Guo, "Local path optimization in the latent space using learned distance gradient," in *2025 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS)*. IEEE, 2025, pp. 15 940–15 946.
- [32] I. A. Şucan and S. Chitta, "Motion planning with constraints using configuration space approximations," in *2012 IEEE/RSJ International Conference on Intelligent Robots and Systems*. IEEE, 2012, pp. 1904–1910.
- [33] J. J. Kuffner and S. M. LaValle, "Rrt-connect: An efficient approach to single-query path planning," in *Proceedings 2000 ICRA. Millennium conference. IEEE international conference on robotics and automation. Symposia proceedings (Cat. No. 00CH37065)*, vol. 2. IEEE, 2000, pp. 995–1001.



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